Comments on error analysis

1) It is not easy to compare the data provided by different manufacturers.
The accuracy and repeatability of a thickness determination depends upon the thickness, the angle of incidence, the type of instrument, the wavelength, etc. I have tried to get some generic determinations of the accuracy for the different designs of ellipsometer.
Often manufacturers give no data! When there is data for repeatability, the layer thickness for which these were measured may not be given. repeatability and accuracy are very different, but sometimes both are not given, etc.

2) I have estimated both the systematic and random errors that one is likely to have for a null ellipsometer, a rotating element ellipsometer, and a birefringence modulation ellipsometer. There is an exact analytic solution for the two quantities refractive index and thickness of transparent layers on a transparent substrate. This starts with data for the complex reflectivity ratio

$$r = \mathrm{Re}(r) + i\,\mathrm{Im}(r) = \tan\Psi e^{i\Delta}$$

and the angle of incidence θ. So from the measured quantity and both its systematic and random error I have deduced Re(r), Im(r) and the errors in these parameters, to give an estimate for δn and δt. I have done this for a layer which is 10nm thick.

3) As the thickness of the layer decreases one becomes unable to separately deduce both n and t. A measure of the minimum thickness t_o below which there is no separation is given by

$$\delta\mathrm{Re} \sim d_o^2$$

for a silicon dioxide layer on silicon at the Brewster angle. $d = 2\pi t/\lambda$.

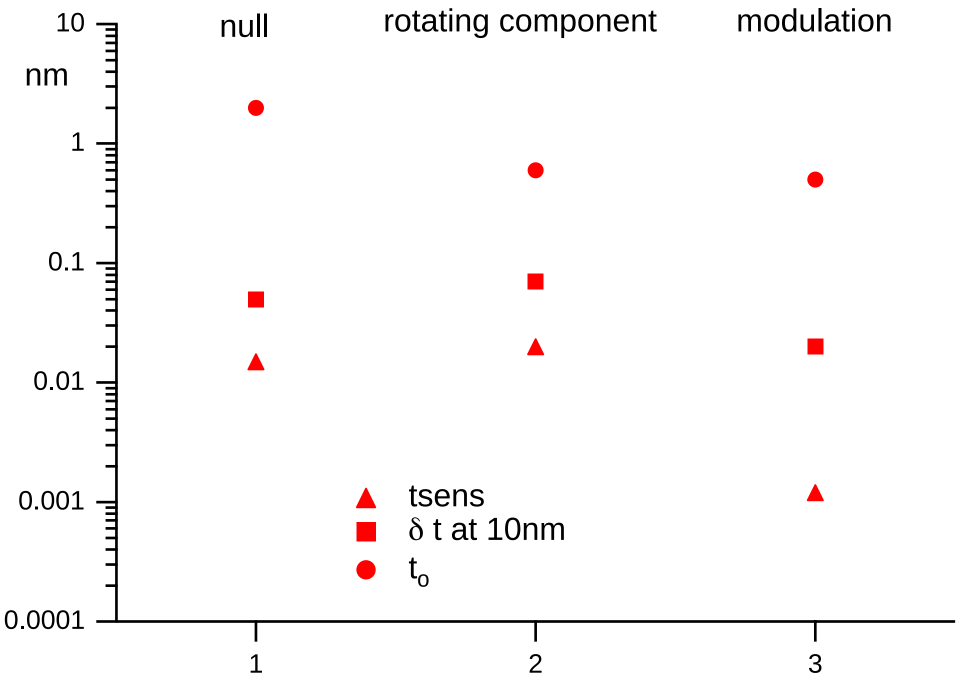
4) Even though there is no separation of n and d at thicknesses below d_o, the instrument may yield an accurate measurement of Im(r), which for transparent substrates on (effectively) transparent silicon is linear in thickness

$$\delta\mathrm{Im} \sim d_{sens}.$$

d_{sens} gives a measure of the ultimate sensitivity of the instrument.

5) For thin layers data taken at different wavelengths provide multiple values for $\delta\mathrm{Im}$ which can provide a separate determination of both t and refractive index. See also Appn.Thin_Layer

The figure below shows our estimates for t_{sens} (from d_{sens}), the accuracy in thickness for a 10nm layer δt, and t_o, the thickness when the error in Re prevents the separate determinations of n and t.



null
rotating component
modulation
nm
10
1
0.1
0.01
0.001
0.0001
1
2
3
tsens
δ t at 10nm
t_o

Commercial ellipsometers

The plot below shows $1/t_{sens}$ for ellipsometers described on the web. These are grouped into three classes, null ellipsometers, rotating analyser-compensator ellipsometers, and modulation ellipsometers. We have defined a Quality factor Q which is equal to the inverse of the thickness sensitivity.
The red points are values I have calculated from the expected sensitivities for each class. The triangles are values taken from the manufacturer's web sites. When these differ significantly from the red points, the manufacturer's claim need to be treated with caution. The triangle under modulation with a quality factor of $1000\mathrm{nm}^{-1}$ is the Beaglehole Instruments Picometer.