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Technical Note 2

Ellipsometry Parameters

Most of the researchers using ellipsometry techniques describe their results in terms of the parameters Ψ and Δ. These parameters are directly related to settings of the analyser and polariser in a null ellipsometer, but are not intrinsically fundemental. They are related to the fundemental quantity r, a complex quantity with real and imaginary parts r_r and r_i by the relation

$$r = r_p / r_s = r_r + i r_i = \tan\Psi\, e^{i\Delta} = \rho\, e^{i\Delta} \ .$$

$\Delta = \delta_p - \delta_s$ is the phase difference between the p and s waves, while tan Ψ is the amplitude of r. Thus

$$\rho = \tan\Psi, \quad r_r = \rho\cos\Delta, \quad r_i = \rho\sin\Delta \, .$$

We believe that ellipsometer researchers would simplify their life considerably if they used r_r and r_i as the significant parameters when describing and reporting their data.

Consider, as an example, the locus of r when a layer of varying thickness is placed on a substrate. This is a commonly occuring situation. The locus in the complex plane of r is close to an ellipse, but is of a quite different shape when the locus of Ψ versus Δ, or alternatively of tan Ψ versus Δ, are shown.

Two examples are given below. In the first we model a protein layer on water, in the second an oxide layer on silicon. Figure 1 shows how r depends on layer thickness for each of the two examples. For both we have chosen an angle of incidence equal to the Brewster angle where $r_r = 0$ for zero layer thickness. Water is a transparent substrate so that r_i also equals zero at the Brewster angle. Silicon has a small imaginary component to its dielectric constant in the infrared, so again r_i starts from very near zero. Figures 2 and 3 show the plots of r_i vs r_r and Ψ vs Δ, respectively. In neither case does Ψ come really close to π, so that Ψ and tan Ψ have a similar behaviour and we only show Ψ versus Δ

Note that Δ changes rapidly from π to $-\pi$ near the origin, giving the semblance that this is a special point on the locus. In fact it is just part of a smooth curve having no inherent singularity associated with it.

The properties of the layer locus will be discussed further in another Technical Note, but in passing it should also be noted that the locus of r in the complex plane becomes exactly a circle when the layer is self-supporting (the incident medium and the substrate have the same dielectric constant).

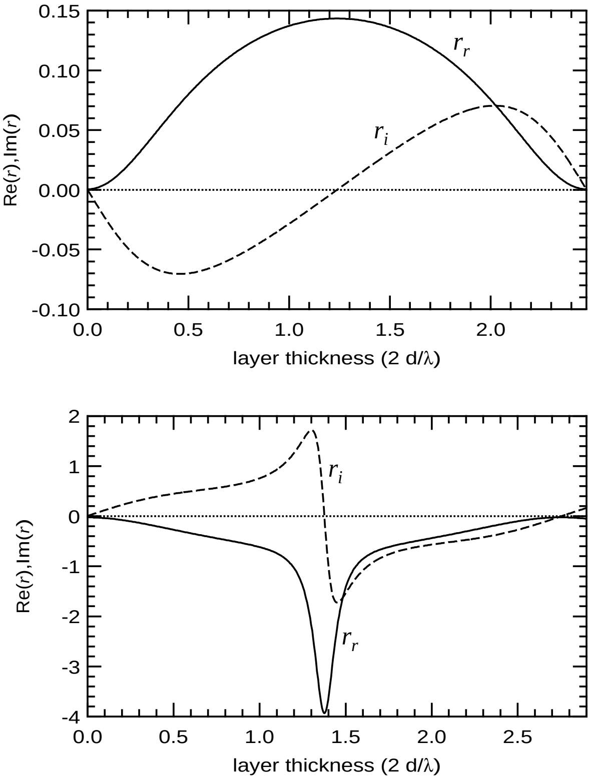
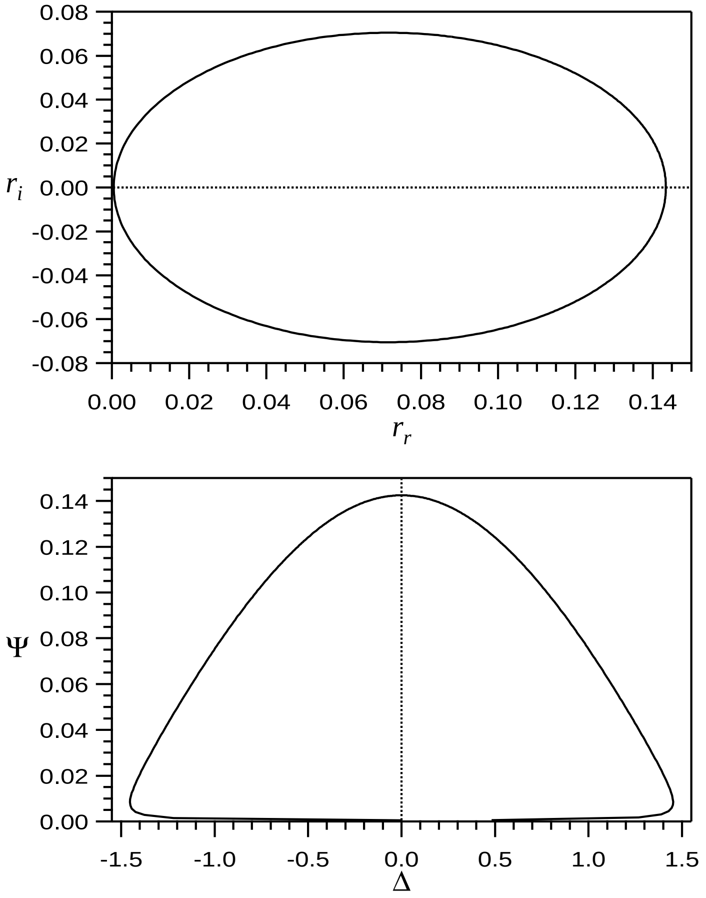


Figure 1: These two figures show r_r and r_i for a protein layer on water (above), and an oxide layer on silicon (below). In both cases $\varepsilon = 2.25$, for water $\varepsilon_2 = 1.78$, for silicon $\varepsilon_2 = 19.1 + i\,0.5$, and the angles of incidence are 53.13 and 77.65 degrees. The thickness is shown in the dimensionless units $2\pi d / \lambda$..

Protein on Water

Figure 2: The locus of r_i *versus* r_r *(above) and* Ψ *versus* Δ *(below), for the case of protein on water. Note how the smooth curve around* $r_i = 0$ *is distorted to become a rapid variation in* Δ*.*



Oxide on Silicon

Figure 3: The locus of r_i *versus* r_r *(above) and of* Ψ *versus* Δ *(below), for an oxide layer on silicon.*